Simple Model for Power Consumption in Aerated Vessels Stirred by Rushton Disc Turbines

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Accurate prediction of power consumption of impeller in an aerated vessel is crucial for a successful scale-up, operation, and design of reactors in general. A mechanistic model with a correlation that can accurately predict power consumption was developed. Taking into account both geometrical parameters and physical properties of the working fluids, this model reduces the number of adjustable parameters better than published empirical relations. It has the advantage of allowing the user to easily scale up agitated vessels and to evaluate power consumption even for large-scale equipment. It has been tested only for Rushton turbines, however, because it is simply based on mass balances, energy balances and continuity equations, it could also be implemented for use with other impellers with very few modifications.

Introduction

Accurate estimation of power consumption in an agitated vessel is an important factor in the scale-up, operation, and design of reactors. The transfer of power from the impeller to the fluid is greatly influenced by aeration, and the power drawn on gassing is usually lower than that under an ungassed condition. This power reduction is due to the formation of cavities behind the blades and to the different density of the fluid under gassed and ungassed conditions (Sensel et al., 1993).

The formation of cavities depends both on fluid properties and on operating conditions. Notwithstanding the complex phenomena, most of the published equations suggest computing the ratio between gassed and ungassed power as a function of the aeration number Fl_G. As pointed out by Tatterson (1991), this dimensionless number has to be used cautiously because a low aeration number can mean a low gassing flow rate, as well as high impeller speed. These two working conditions present distinctly different states in the resulting mixtures. These effects, together with changes induced by the physical properties of the working fluids, imply that a general universal power correlation has not yet been obtained.

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The study of the formation of gas cavities for radial flow Rushton turbines has been performed by several authors (Warmoeskerken and Smith, 1985; Smith and Warmoeskerken, 1985; Nienow et al., 1985). They reported that "vortex" cavities are formed at a given impeller speed and low gas-flow rates. When the gas-flow rate is increased, these evolve into "clinging" cavities. Three "large" smooth cavities are then formed giving rise to the so-called "3-3" structure. At still higher gas-flow rates, "bridging" cavities may be formed and ultimately, "ragged" cavities are formed and the impeller is said to be "flooded." They highlighted that the formation of cavities behind the impeller blades affected the power drawn from the impeller in the aerated situation.

Therefore, to describe the fluid dynamic of gas-liquid stirred-tank reactors properly, it is necessary to look for the manner in which the cavities interfere with the transfer of power and how they can change the prevailing transfer mechanism. Finally, to solve the problem of power transfer, it is necessary to evaluate the origin of gas arriving at the impeller properly. This gas consists of the gas arriving through the sparger and a recirculated portion. The amount of recirculated gas can significantly affect the power requirement in a stirrer vessel, but, unfortunately, very few studies on gas recirculation have been published, probably because accurate measurements are rather difficult to perform.

Much experimental data on power input have been published for single turbine stirred-tank reactors, but, unfortunately, there is not the same availability of data for multiple impeller systems, which are more interesting from an industrial point of view. Hudcova et al. (1989) measured the power drawn by each impeller separately for a two impeller configuration, by strain gauging and related the power to the visualized flow pattern changes for different impeller spacings. They reported that the lower impeller could be treated as a single aerated impeller operating alone when the spacing between it and the one above was about one vessel diameter. This experimental observation partially agrees with the experimental data by Linek et al. (1996), who showed that the adjacent impellers do not interfere when they are located at a distance of more than double their diameter. Earlier, Hicks and Gates (1976) and Nienow and Lilly (1979) and, subsequently, Abrardi et al. (1990), observed a similar dependency in their works.

Therefore, if the clearance between turbines is sufficiently large, the power consumption for the bottom impeller can be analyzed in the same way as that for the single impeller system. According to Hudcova et al. (1989), however, the power drawn by the upper impeller has to be analyzed carefully because it is greater than that drawn by the bottom impeller.

Model to Simulate Aerated Stirred Tank Behavior

In order to evaluate the aerated power number, it is necessary to know the exact gas-flow rate into the turbines. Therefore, first of all, it is necessary to evaluate the recirculation of the gas and then the aerated power number for single and multiple impellers can be evaluated starting from the mass balance.

Gas recirculation for a single impeller

The present model for the evaluation of the recirculated gas assumes that the two-phase mixture in the stirred tank is homogeneous, in other words, that there is no gradient of the gas holdup throughout the vessel. This is a simplifying hypothesis because gas holdup gradients are in fact present in stirred-tank reactors, especially close to the turbine, but it allows a simple model to be formulated, which avoids complex fluid dynamic assumptions that imply long computation time.

Moreover, the flow pattern in stirred-tank reactors is a function of the geometry of the vessel and of the turbine. Different flow regimes are possible; for instance, when dual turbines are used, Mahmoudi and Yianneskis (1992) identified three different flow patterns: diverging, merging and parallel flow. Therefore, an accurate description of the fluid dynamics of the vessel should take into account the flow regimes as well. Unfortunately, this implies complex fluid dynamic assumptions, so in the present work the turbine fluid dynamics has been simplified as shown in Figure 1 for a single turbine and in Figure 2 for a dual impeller system.

According to the two assumptions made above, when a single turbine is analyzed, if the liquid velocity close to the turbine V_l is less than the bubble raising velocity V_b, all bubbles present move to the top of the vessel and recirculation is absent. Otherwise, if the liquid velocity overcomes the bubble rising velocity, the gas recirculation flow rate is proportional to the gas holdup ϵ and to the effective gas velocity close to the turbine V_g. Accordingly, the flow rate of recirculated gas Q_r which returns into the turbine together with freshly sparged gas, can be evaluated as

$$\begin{cases} Q_r = A \cdot \epsilon \cdot V_g = A \cdot \epsilon \cdot (V_l - V_b) & \text{if } V_l > V_b \text{ (a)} \\ Q_r = 0 & \text{if } V_l \leq V_b \text{ (b)} \end{cases}, \quad (1)$$

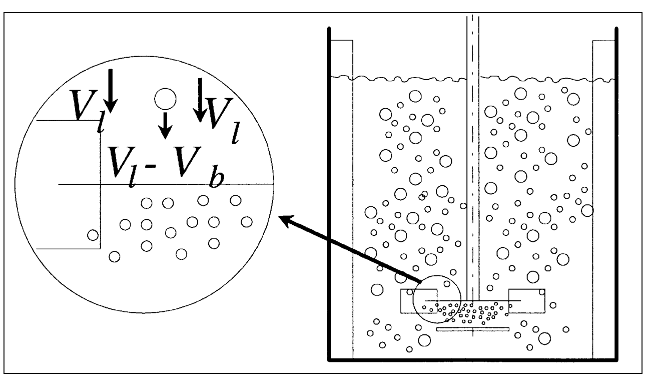

Figure 1. Bubbles and liquid motion in a stirred tank with a single impeller.

The bubble velocity vector and the liquid velocity vector close to the turbine are shown.

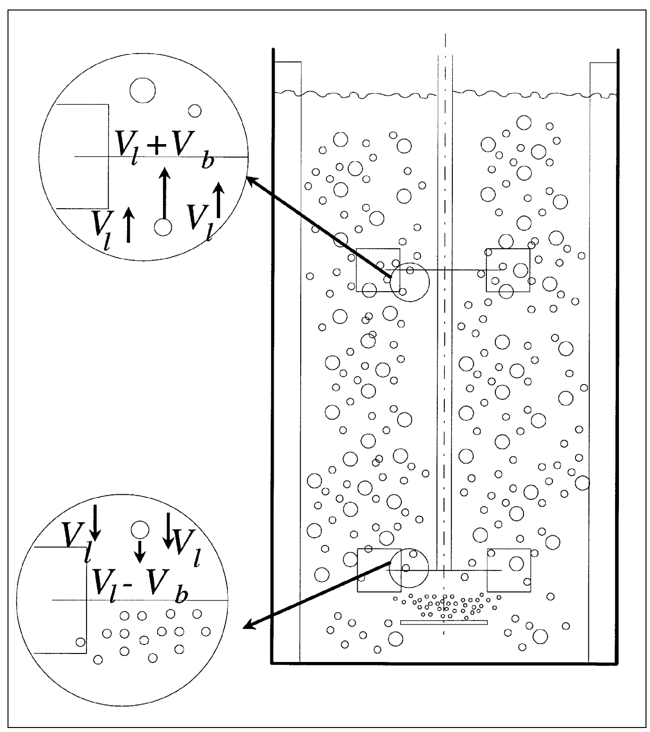

Figure 2. Bubbles and liquid motion in a stirred tank with multiple impellers.

The bubble velocity vector and the liquid velocity vector close to the turbines are shown.

where A is the cross section for the recirculation and has been defined as

$$A = \left(\frac{D}{2}\right)^2 \cdot \pi. \quad (2)$$

The effective liquid and gas velocities of the mixture flowing into the turbine V_l and V_g can be computed from the following continuity equation as function of the mean velocity V_m and of the bubble rising velocity V_b

$$V_m = V_l \cdot (1-\epsilon) + V_g \cdot \epsilon, \quad (3)$$

where

$$V_g = V_l - V_b. \quad (4)$$

Finally, the equations necessary to evaluate the liquid and gas velocities can be rewritten as

$$V_l = V_m + V_b \cdot \epsilon, \quad (5)$$

$$V_g = V_m - V_b \cdot (1-\epsilon). \quad (6)$$

The mean velocity can be expressed as

$$V_m = \frac{Q_m}{A}, \quad (7)$$

where the mixture flow rate Q_m is proportional to the stirrer speed N and to D^3

$$Q_m = Fl \cdot N \cdot D^3, \quad (8)$$

where Fl is the flow number. Finally, Eq. 1a can be rewritten as

$$Q_r = A \cdot \epsilon \cdot \left(Fl \cdot \frac{N \cdot D^3}{A} - V_b \cdot (1-\epsilon)\right) = Fl \cdot \epsilon \cdot N \cdot D^3 - V_b \cdot \frac{D^2}{4} \cdot \pi \epsilon \cdot (1-\epsilon). \quad (9)$$

The ratio between the recirculated gas-flow rate and the gas-flow rate coming from the sparger assumes the final form

$$\frac{Q_r}{Q_{gv}} = Fl \cdot \epsilon \cdot \frac{N \cdot D^3}{Q_{gv}} - \frac{V_b \cdot D^2 \cdot \pi \cdot \epsilon \cdot (1-\epsilon)}{4 \cdot Q_{gv}}. \quad (10)$$

Gas distribution for a multiple impeller system

According to Hudcova et al. (1989), the bottom impeller in multiple impeller systems can be treated as a single impeller if the clearance from the bottom is sufficient and if the spacing between the impellers is larger than one vessel diameter. Under these assumptions, it is therefore reasonable to assume that the total gas-flow rate arriving at the lower impeller Q_l can be evaluated as

$$Q_l = Q_{gv} + Q_r = Q_{gv} + Fl \cdot N \cdot D^3 \cdot \epsilon - \frac{D^2}{4} \cdot \pi \cdot \epsilon \cdot V_b \cdot (1-\epsilon). \quad (11)$$

For the subsequent impellers, the gas-flow rate arriving at the impeller, Q_u is different from the gas-flow rate Q_l, flowing into the lower impeller. This is due to the fact that all the gas from the sparger goes through the lower impeller, but only a part of it passes through the other impellers. In fact, due to the recirculation induced by the subsequent impellers, only a part of the gas flowing to the liquid surface passes inside the impellers while flowing to the liquid surface. The gas-flow rate arriving to the subsequent impellers can be computed as shown for the recirculated gas-flow rate for the bottom impeller (Eq. 9). The only difference is that the gas velocity has to be evaluated as the sum of V_b and V_l (see Figure 2), it follows that

$$V_l = V_m - V_b \cdot \epsilon, \quad (12)$$

$$V_g = V_m + V_b \cdot (1-\epsilon), \quad (13)$$

therefore

$$Q_u = A \cdot \epsilon \cdot [V_m + V_b \cdot (1-\epsilon)] = Fl \cdot \epsilon \cdot N \cdot D^3 + \frac{D^2}{4} \cdot \pi \cdot \epsilon \cdot V_b \cdot (1-\epsilon). \quad (14)$$

Finally, the ratio between the gas flowing into the bottom and the subsequent impellers can be evaluated as

$$\frac{Q_l}{Q_u} = \frac{Q_{gv} + Fl \cdot N \cdot D^3 \cdot \epsilon - \frac{\pi \cdot D^2}{4} \cdot \epsilon \cdot V_b \cdot (1-\epsilon)}{Fl \cdot N \cdot D^3 \cdot \epsilon + \frac{\pi \cdot D^2}{4} \cdot \epsilon \cdot V_b \cdot (1-\epsilon)}. \quad (15)$$

Power consumption

It is well known that the power consumption in an aerated-agitated vessel is proportional to the fluid-flow rate swept by the turbine and to the specific energy of the fluid moved by the turbine. Thus, it is possible to compute the power requirement in gas-liquid agitated vessel in the following way

$$P_g = \frac{Q_m \cdot \Delta M \cdot \rho_t}{\mu_{mbc}}, \quad (16)$$

where Q_m is the volumetric fluid flow rate defined by Eq. 8, ρ_t is the density of the fluid inside the blades of the turbine, ΔM is the specific energy, and μ_{mbc} is the mechanical efficiency. In the ungassed condition, the power can be computed in a similar way

$$P_o = \frac{Q_m \cdot \Delta M \cdot \rho_l}{\mu_{mbc}}, \tag{17}$$

where ΔM is proportional to $(N \cdot D)^2$ and Q_m is proportional to $(N \cdot D^3)$ and neither of them are functions of the fluid properties over a wide range of operating conditions; thus, it can be assumed that

$$\frac{P_g}{P_o} = \frac{\rho_t}{\rho_l}. \tag{18}$$

Finally, to evaluate the ratio between ungassed and gassed power consumption, it is necessary to evaluate the density of the fluid inside the turbine. The mean density of the fluid inside the turbine ρ_t depends upon the dimension of the cavities behind the blades and upon the density of the gas-liquid mixture swept by the turbine. Defining the gas holdup of cavities at the disc plane β_t as the following

$$\beta_t = \frac{\text{Area of the cavities}}{\text{Area inside the blades of the turbine}}, \tag{19}$$

it is possible to compute the mean density of the mixture ρ_t as "seen" by the turbine as

$$\rho_t = \rho_m \cdot (1 - \beta_t) + \rho_g \cdot \beta_t, \tag{20}$$

where ρ_g and ρ_m are the gas density and the mean density of the fluid flowing into the turbine, respectively. Neglecting the slip velocity between the gas and liquid phases driven by the turbine, it is possible to evaluate the gas holdup of the gas/liquid mixture into the turbine ϵ_m as

$$\epsilon_m = \frac{Q_{gv} + Q_r}{Q_m}, \tag{21}$$

and therefore

$$\rho_m = (1 - \epsilon_m) \cdot \rho_l + \epsilon_m \cdot \rho_g. \tag{22}$$

From the previous assumption, it derives that

$$\frac{P_g}{P_o} = (1 - \epsilon_m) \cdot (1 - \beta_t) + \frac{\rho_g}{\rho_l} \cdot \left[\epsilon_m + \beta_t \cdot (1 - \epsilon_m) \right]. \tag{23}$$

This equation can be used to predict the power consumption when the turbine is not flooded. The same approach can also be used for the upper turbines. The only difference is in the gas holdup ϵ_m for these turbines simply becomes

$$\epsilon_m = \frac{Q_u}{Q_m}. \tag{24}$$

Experimental Studies

The vessel used was constructed from Perspex to enable visual observations (vessel diameter $T = 0.72$ m equipped with four $0.1T$ baffles). One or two 6-bladed disc turbines ($D/T = 1/3$) were used with an aspect ratio of $H/T = 1$ or 2. The clearance of the impeller from the tank base was set at $T/3$ and the distance between two impellers ΔC on the shaft was T. All experimental work was performed using air and tap water. Air was introduced from a ring sparger from the bottom of the vessel. The power of the individual impeller on the shaft was measured using strain gauges. A digital revolution counter monitored the speed.

From an experimental point of view, gas recirculation rate can be evaluated in two different ways: by measuring the absorbed power (van't Riet et al., 1976; Nienow et al., 1977) or using a tracer (Nienow et al., 1979; Ezimora and Lubbert, 2000). In the present article, the power method suggested by Nienow et al. (1977) was used for both the single turbine and the dual impeller systems.

Discussion and Analysis of the Experimental Data

Before starting with the analysis of the experimental data, it is necessary to declare the hypothesis used in the present model and what differences exist between it and all the other models published so far. Until now, all published models have introduced many different adjustable parameters to evaluate gas recirculation rate and power number under a gassed condition in order to obtain an acceptable degree of convergence between theories and experiments. Notwithstanding this introduction of many adjustable parameters, the proposed theories only permit the accurate prediction of the experiments used to fit the models, whereas large errors arise when the comparison is made between sets of experiments performed by different authors. The purpose of the present work is to suggest a mechanistic model that can reduce the number of the necessary adjustable parameters that can be used to scale up reactors.

Therefore, in order to close the model, it is necessary to know gas holdup ϵ and to evaluate the flow number Fl, the bubble rising velocity V_b, and the cavities gas holdup β_t.

In the present work to evaluate gas holdup ϵ, the equation suggested by Paglianti et al. (2000) has been used, whereas the flow number for a Rushton turbine has been assumed equal to 0.75, as suggested by Revill (1982).

To compute the bubble rising velocity, which depends on the physical properties of the working fluids, it is necessary to assume a typical bubble size.

Van Dierendonck et al. (1968) showed that in nonionic systems, if the stirred speed is sufficiently high, the influence of agitation stabilizes the system and the bubble diameter becomes a function of the Eotvos number Eo_{crit}

$$d_b = \left[\frac{Eo_{\text{crit}} \cdot \sigma}{(\rho_l - \rho_g) \cdot g} \right]^{0.5} \tag{25}$$

Comparing experimental data by Van Dierendonck et al. (1968) with the analysis by Clift et al. (1978) on single bubble behavior, it emerges that the value of Eo_{crit}, suggested by Van Dierendonck et al. (1968), corresponds fairly well to the transition from a spherical to ellipsoidal/wobbling regime

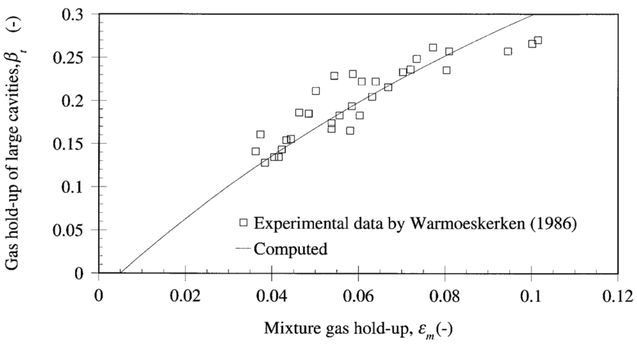

Figure 3. Cavities gas holdup.

Comparison between experimental data by Warmoeskerken (1986) and the present relation. (Experimental data related to the "3-3" structure.)

suggested by Clift et al. (1978). Therefore, in the present work, the bubble size has been evaluated as the dimension corresponding to the transition from spherical to ellipsoidal/wobbling regime. It is necessary to point out that, according to the Clift et al. (1978) analysis, Eo_{crit} is not a constant value, but is a function of the property group M

$$M = \frac{g \cdot \mu_l^4 \cdot (\rho_l - \rho_g)}{\rho_l^2 \cdot \sigma^3}. \quad (26)$$

Finally, from the knowledge of M and using the Clift et al. (1978) analysis, it is possible to evaluate bubble size and bubble rising velocity.

Therefore, only the cavities gas holdup β_t has to be evaluated to close the model.

Many articles have been published on the influence of the cavities structure on power consumption, but unfortunately few experimental data have been published on the size of the cavities. It is clear that their size depends on the gas-flow rate flowing into the turbine, but, to our knowledge, no equation has been published so far.

Figure 3 shows the experimental data of cavities gas holdup by Warmoeskerken (1986), related to the 3-3 structure, as a function of the gas holdup of the mixture in the turbine ϵ_m, computed using Eq. 21. The figure also shows that, assuming an equal size for each large cavity and neglecting the presence of clinging cavities, the maximum gas holdup in the cavities is about 0.25 when the 3-3 structure is present. Therefore, in the present work, it has been assumed that the maximum gas holdup in the cavities is equal to 0.5 when a 6 large cavities regime occurs. This assumption agrees with the experimental measurements by Saito and Kamiwano (1988), who measured about 0.5 as the maximum gas void fraction close to a Rushton turbine. Finally, the figure shows that cavities gas holdup β_t can be evaluated as a function of ϵ_m. So, by fitting the experimental data by Warmoeskerken (1986), the following equation has been used in the present work

$$\begin{cases} \beta_t = \dfrac{\epsilon_m - 0.005}{\epsilon_m + 0.218} & \text{if} \quad \epsilon_t \leq 0.228 \\ \beta_t = 0.5 & \text{if} \quad \epsilon_t > 0.028 \end{cases}, \quad (27)$$

and it is plotted in Figure 3 as a continuous line.

Before comparing experimental and computed data, it is necessary to analyze the weight of the two main simplifying assumptions used in the model: the homogeneous gas distribution and the influence of the flow patterns.

Figure 4 shows the ratio of gassed on ungassed power input as a function of the gas-flow number Fl_G, if a single Rushton turbine is used. The continuous line shows the computed values when the homogeneous conditions occur, whereas the dashed line shows the values computed assuming that the gas holdup is 50% greater than the value at homogeneous conditions around the turbine.

Since the goal of this work is to propose a model which does not introduce unnecessary adjustable parameters, the influence of gas holdup gradients has been neglected, even if this can cause small changes in the computed power input.

The other simplifying hypothesis regards the flow field around the turbines. When analyzing a single impeller or the bottom impeller of a multiple impeller system, the influence of the loop below the turbine has been neglected, whereas for the subsequent impellers in multiple impeller systems, the influence of the loop above the turbine has been neglected. Figure 5 shows the ratio of gassed on ungassed power input as a function of the gas-flow number Fl_G, when a single turbine, or the bottom one in multiple impeller systems, is analyzed. The continuous line shows this model and the dashed line shows the computed results when only the lower loop is taken into account. The real behavior of the turbine is between these two curves and the exact position depends on the clearances between the analyzed turbine, the bottom of the tank, the upper turbine, and the liquid layer.

Finally, it was decided to neglect the influence of flow patterns both in the analysis of a single turbine and in multiple impeller systems. This can induce some errors, especially when multiple impeller systems are analyzed, but it permits the mechanistic model to be closed, while avoiding the introduction of any adjustable parameters.

Recirculation for a single impeller system

As pointed out by several authors (Nienow et al., 1979; Takenaka and Takahashi, 1996; van't Riet et al. 1976), the recirculation rate is a very important parameter for the design of agitators for gas-liquid contacting. Few articles have been published on this topic despite its importance in both power consumption and mass-transfer phenomena.

Little experimental data has been presented in literature and, to our knowledge, only the equation by Takenaka and Takahashi (1996) is available in open literature. Figure 6a shows the comparison between some experimental data available and the values computed with the present model. Experimental data by van't Riet et al. (1976), Roustan (1985), Ezimora and Lubbert (2000) and Nienow et al. (1977) have been analyzed according to Nienow et al. (1977). It should be noted that the tank diameter was varied in the range 0.29−0.91m and the ratio D/T in the range 0.33−0.5.

The comparison shows that the present relation can be used successfully, and obtains accurate predictions for all of the published sets of data with the only exception of the data by Nienow et al. (1979) and of a set of data by van't Riet et al. (1976). This large difference between the computed and ex-

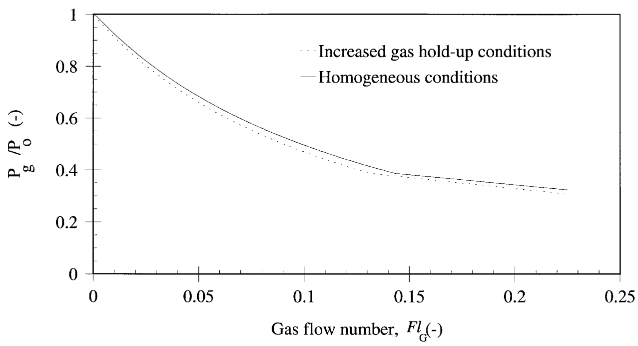

Figure 4. Power consumption in an aerated medium: single impeller system, gas gradient effect.

Water-Air system, $D = 0.24$ m, $T = 0.72$ m, $N = 2.83$ rps.

perimental data by Nienow et al. (1979) is probably due to the sparger tracer used in that work. In fact the sparger used distributed the tracer equally around the circumference of the impeller. In this way, the tracer does not flow into the turbine and, therefore, the traced bubbles are greater than the bubbles coming out from the turbines. If this is true, the traced bubbles present a higher rising velocity than the others and they would consequently recirculate with greater difficulty. This observation could also explain the apparent discrepancy of this particular data set with the previous work of Nienow et al. (1977), as evidenced by the authors, and seen in this present analysis. The comparison shows that present relation only predicts properly a part of the experimental data by van't Riet et al. (1976). If the set of data related to the larger tank diameter ($T = 440$ mm) is used, the relative mean square error is smaller than that associated with the smaller tank, $T = 190$ mm. This difference is probably due to the experimental setup used. In fact, to evaluate the recirculated gas-flow rate for the smaller vessel, the power consumption without recirculation was evaluated using a square vessel instead of a cylindrical one.

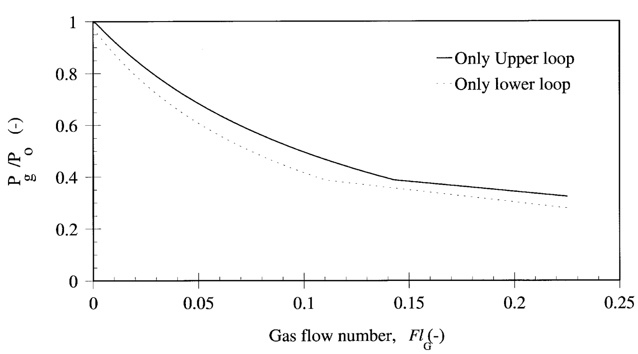

Figure 5. Power consumption in an aerated medium: single impeller system, flow pattern effect.

Water-Air system, $D = 0.24$ m, $T = 0.72$ m, $N = 2.83$ rps.

Figure 6b shows the comparison between available experimental data and the equation proposed by Takenaka and Takahashi (1996). The accuracy obtained using the latter equation is lower than that obtained with the former, especially for large tanks.

Gas distribution for multiple impeller system

Less data is available for the evaluation of the behavior of a multiple impeller system compared to that available for single impeller systems. To our knowledge, the only works available are by Hudcova et al. (1989), Machon et al., (1985), and Warmoskerken and Smith (1985). One of the most important experimental observations which must be highlighted is that if multiple turbines are used, the power consumed by each impeller (except the bottom one) is the same (see Warmoeskerken and Smith (1988), Hicks and Gates (1976) and Nienow and Lilly (1979)). This behavior is also predicted well by the theoretical analysis suggested in this article.

The present model can only be strictly used if the clearance between the two impellers ΔC is sufficient to consider them as fully separated; in other words, it can not be used when merging flow occurs. As suggested by Hudcova et al. (1989), it is possible to assume that this condition is achieved at the ratio $\Delta C/D = 3$.

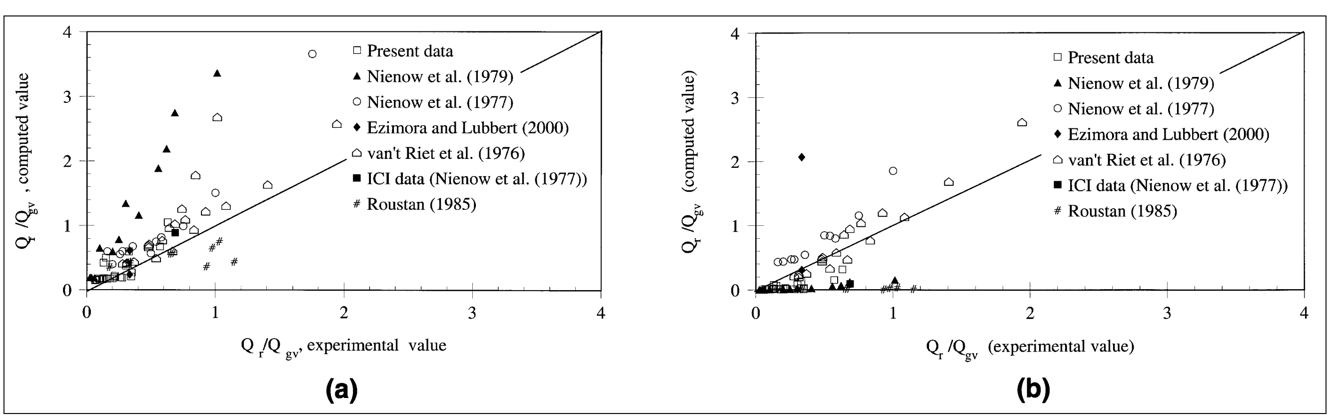

Figure 6. Recirculated gas flow: single impeller system.

Comparison between experimental and computed measurements of Q_r/Q_{gv}. (a) Present model; (b) Takenaka and Takahashi (1996) relation.

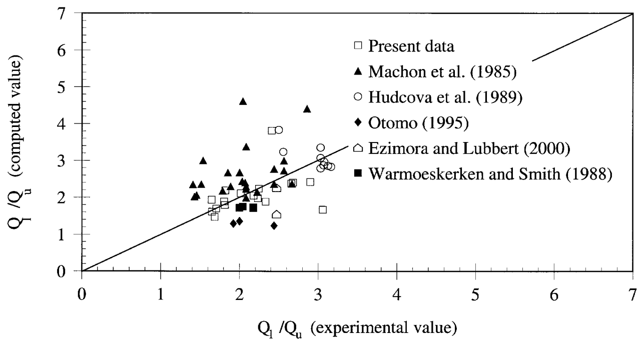

Figure 7. Recirculated gas flow: dual impeller system.
Comparison between experimental and computed measurements of Q_l/Q_u.

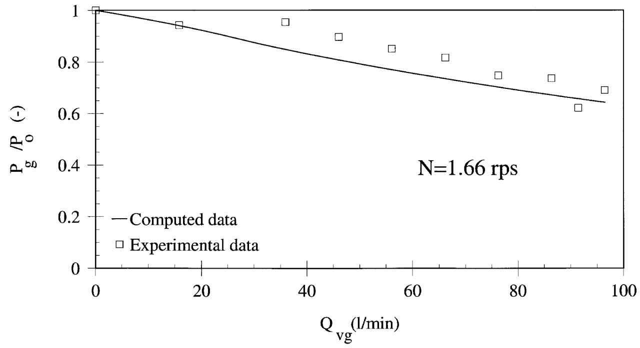

Figure 8. Power consumption in aerated medium: single impeller system.
Comparison between experimental and computed measurements. (present data, water-air system, $D = 0.24$ m, $T = 0.72$ m, $N = 1.66$ rps.)

Figure 7 shows the comparison between experimental and computed data. The scatter between experimental and computed data is high, with a relative mean-square error of 34%, but it is necessary to point out that the tank diameter was varied in the range 0.3–0.72m and the D/T ratio was varied in the range 0.33–0.5. It must also be noted that no other relation is available to predict the gas distribution for dual impeller systems.

Power consumption

Single Impeller. Many articles have already been published that suggest a way of computing the power consumption in an aerated medium. Visual observation shows that the power consumption decreases with increasing gas-flow rate. Until now, only some empirical relations with uncertain ranges of applicability have been published (Hughmark, 1980; Nagata, 1975; Pharamond et al., 1975; Calderbank, 1958; Cui et al., 1996). These authors suggest different equations in which the ratio P_g/P_o has been expressed as a function of different dimensionless numbers. As pointed out by Midoux and Charpentier (1984) and by Greaves and Barigou (1988), the agreement between the available equations is absolutely insufficient. Figures 8 to 10 show the comparison between the computed and the experimental data obtained in present work. The accuracy of the prediction seems to be sufficient even if some systematic errors occur when experimental data at $N = 2.83$ rps are analyzed.

Some experimental data are available in literature on the power consumption for six blade Rushton turbines. This article analyzes both the data obtained for this work together with other sets of data. The large amount of data available made it possible to study both the influence of the tank diameter and of the D/T ratio. Table 1 shows the list of the literature data analyzed with a total number of 422 measurements for single impellers systems used in this comparison.

Figure 11 shows the comparison between experimental data and the present model. The results obtained with the present model agree with the most widely used empirical relations published so far, as can be seen by analysis of Table 2.

The table shows that, apart from the partial exception of the relation suggested by Hughmark (1980), all the others can be used obtaining a similar degree of accuracy.

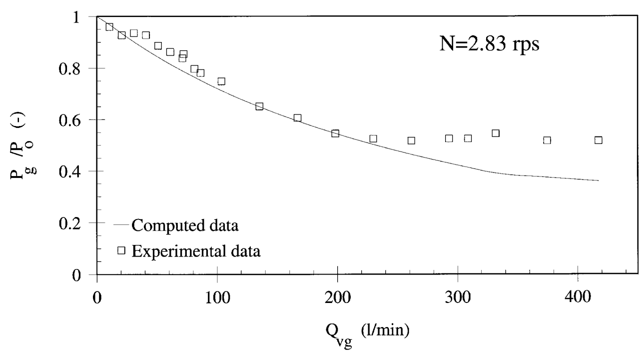

Figure 9. Power consumption in aerated medium: single impeller system.
Comparison between experimental and computed measurements. (Present data, water-air system, $D = 0.24$ m, $T = 0.72$ m, $N = 2.83$ rps.)

Multiple Impellers. As shown in the previous section, some empirical relations and some sets of experimental data are available to evaluate the power consumption in gas-liquid

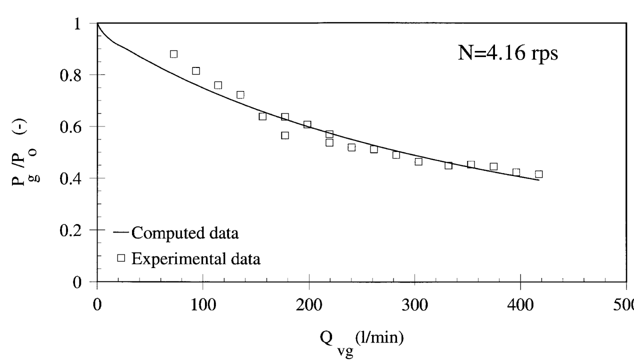

Figure 10. Power consumption in aerated medium: single impeller system.
Comparison between experimental and computed measurements. (Present data, water-air system, $D = 0.24$ m, $T = 0.72$ m, $N = 4.16$ rps.)

Table 1. Experimental Details of Previous Works: Gas-Liquid Reactors Equipped with a Single Rushton Turbine

Authors	T [m]	D/T
van't Riet et al. (1976)	0.19 ÷ 1.5	0.12 ÷ 0.4
Warmoeskerken and Smith (1985)	0.44	0.4
Nienow et al. (1985)	0.44 ÷ 0.61	0.33 ÷ 0.55
Bruijn et al. (1974)	0.44	0.4
Warmoeskerken and Smith (1988)	0.625 ÷ 1.2	0.4
Machon et al. (1985)	0.29	0.33
Roustan (1985)	0.43 ÷ 1	0.33 ÷ 0.66
Nienow et al. (1977)	0.29 ÷ 0.91	0.32 ÷ 0.5
Hudcova et al. (1989)	0.56	0.33

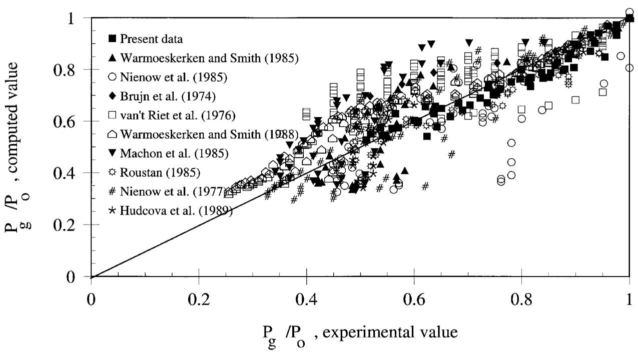

Figure 11. Power consumption in aerated medium: single impeller system.

Comparison between experimental measurements and computed data.

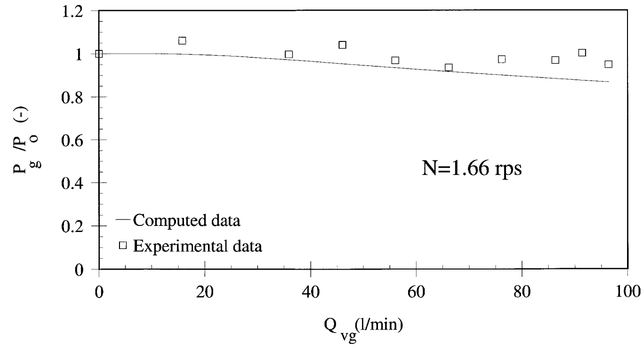

Figure 12. Power consumption in aerated medium: dual impeller system.

Comparison between experimental and computed measurements, upper impeller. (Present data, water-air system, $D = 0.24$ m, $T = 0.72$ m, $N = 1.66$ rps.)

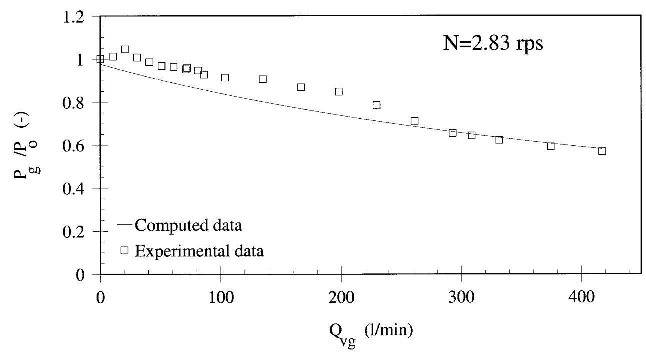

Figure 13. Power consumption in aerated medium: dual impeller system.

Comparison between experimental and computed measurements, upper impeller. (Present data, water-air system, $D = 0.24$ m, $T = 0.72$ m, $N = 2.83$ rps.)

tanks agitated with a single impeller. On the contrary, little data has been published regarding multi-impellers systems even though they are extensively used in industry. As pointed out by Cui et al. (1996) and by Hudcova et al. (1989), the influence of the gas-flow rate on the power requirement by the impellers is more significant for the bottom impeller than for the other impellers. This experimental observation demonstrates that using an empirical relation obtained for a single impeller to evaluate the power uptake by a multiple impeller can induce great errors.

Figures 12 to 14 show the comparison between the computed and the experimental data obtained in the present work. The accuracy seems to be sufficient. Notwithstanding this, it is necessary to highlight that the present model does not contain any adjustable parameters to fit power input.

According to our knowledge, the only available relation for the evaluation of the power uptake to the impeller above the bottom impeller is that suggested by Cui et al. (1996). These authors assumed that the cavities behind the bottom impeller are larger than those behind the upper impellers. For this

Table 2. Present Model vs. Most Widely Used Empirical Relations: Mean-Square Errors, Gas-Liquid Reactors with Single Rushton Turbine

Models	Mean-Square Error
Present	19%
Hughmark (1980)	35%
Nagata (1975)	19%
Pharamond et al. (1975)	18%
Calderbank (1958)	23%
Cui et al. (1996)	20%

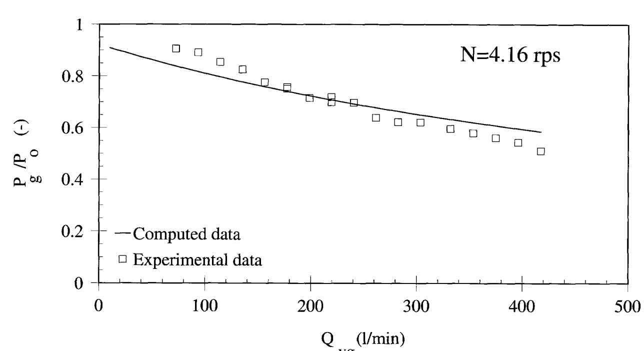

Figure 14. Power consumption in aerated medium: dual impeller system.

Comparison between experimental and computed measurements, upper impeller. (Present data, water-air system, $D = 0.24$ m, $T = 0.72$ m, $N = 4.16$ rps.)

Table 3. Previous Experimental Works: Gas-Liquid Reactors with Multiple Impellers System

Authors	T [m]	D/T
Machon et al. (1985)	0.29	0.33
Warmoeskerken and Smith (1988)	0.625	0.4
Hudcova et al. (1989)	0.56	0.33

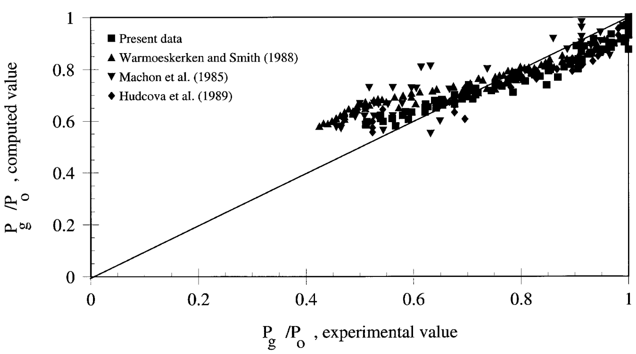

Figure 15. Power consumption in aerated medium: dual impeller system, upper turbine.

Comparison between experimental measurements and computed data with the present model.

reason, they suggested using two different equations for evaluating the power consumption. The present model considers both the impellers using the same theoretical approach and avoids introducing any separate adjustable parameters. Table 3 shows a list of the published experimental data on the power consumption of impellers above the bottom impeller that have been considered in the present work; a total number of 269 experimental points have been analyzed.

Figures 15 and 16 show a comparison between the experimental and the computed data produced by the present model and the Cui et al. (1996) relation, respectively. The latter relation was applied using the parameters K_1, K_2 and K_3 suggested by the authors for the geometry with $D = 0.256$ m and $T = 0.64$ m since there was no alternative to evaluate them for a different geometry. The present model and Cui's model

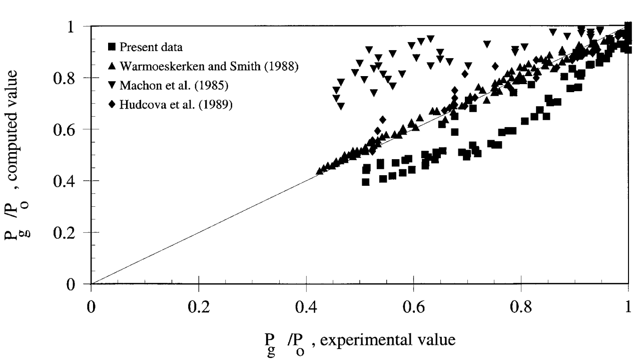

Figure 16. Power consumption in aerated medium: dual impeller system upper turbine.

Comparison between experimental measurements and computed data with the Cui et al. (1996) relation.

Table 4. Previous Experimental Works: Gas-Liquid Reactors with a Multiple Impeller System*

Authors	T [m]	D/T	No. of Impellers
Hudcova et al. (1989)	0.56	0.33	2
Roustan (1985)	0.43 ÷ 0.78	0.33 ÷ 0.46	2–3
Machon et al. (1985)	0.29	0.33	2
Nienow et al. (1994)	1.98	0.35	4

*Total values of power consumption.

induce comparable mean-square errors, 14% and 19%, respectively. Nevertheless, some comments are necessary; the present model (see Figure 15) induces the largest error, especially due to the experimental set by Warmoeskerken and Smith (1985) relative to a measured P_g/P_o in the range 0.4–0.5. This means that for those experimental conditions, the ratio P_g/P_o for the bottom impeller has to be lower than 0.4. If this is true, the bottom impeller is probably flooded and this could be the reason why the present model cannot accurately predict this set of experimental data.

Moreover, the relation by Cui et al. (1996) that was tuned on the Warmoeskerken and Smith (1988) data seems to introduce systematic errors when the geometrical characteristics of the system are changed.

Finally, the present model makes it is possible to analyze very complex systems and even full-scale fermenters (such as the experimental data by Nienow et al. (1994)). The published experimental data analyzed in this section, relating to total power consumption in multiple impeller systems, is shown in Table 4. This includes a total of 390 analyzed data.

Figures 17 and 18 show a comparison between experimental data and computed values using the present model and the Cui et al. (1996) relation. The mean-square errors are 19% and 55%, respectively.

The difference in the mean-square errors is significant, but the most important observation arises from the analysis of the computed results when the data from a full-scale reactor (Nienow et al., 1994) are analyzed. In this case, the present model gives a sufficiently accurate prediction with a mean-square error of about 30%; on the contrary, the relation by

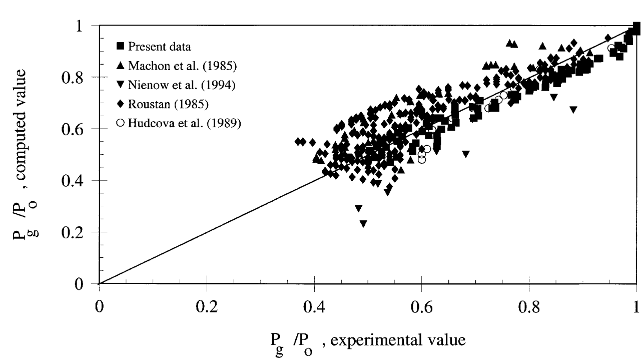

Figure 17. Power consumption in aerated medium: multiple impeller system total value of power input.

Comparison between experimental measurements and computed data with the present model.

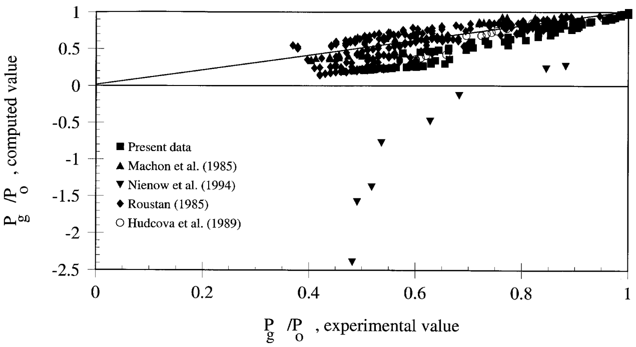

Figure 18. Power consumption in aerated medium: multiple impeller system total value of power input.

Comparison between experimental measurements and computed data with the Cui et al. (1996) relation.

Cui et al. (1996) reports a negative value for the computed P_g/P_o ratio for some of the working conditions, which is a nonsensical result. Analysis of Figure 17 shows that the present model seems to be a useful tool across the full range of working conditions.

Conclusions

A mechanistic model has been suggested to evaluate the parameters that are necessary to properly design an aerated vessel stirred by Rushton disc turbines. The present model is based on simple mass balances, with only the equation suggested by Paglianti et al. (2000) to evaluate mean gas holdup into the stirred tank reactor and an equation to evaluate cavities gas holdup necessary as closure relations. It has to be stressed that, on the contrary to all the relations published so far, no adjustable parameters have been introduced into the present model to fit the power input experimental data. Moreover, the present model the evaluation to be made of gas recirculation and power consumption is not just for single turbine systems, but also for multiple impeller systems.

It can be a useful tool in the design stage because it works properly not just for small-scale vessels, but also for full-scale fermenter reactors.

Due to the nature of the present model, it can be easily modified with a few adjustments for use with reactors equipped with different type of turbines and vessel geometry.

Acknowledgments

The authors would like to thank the staff at the School of Chemical Engineering, University of Birmingham (U.K.) for their patience and support and a special thanks to Prof. A.W. Nienow for the precious suggestions and for his friendship.

This work was financially supported by the "Ministero dell'Universitá e della Ricerca Scientifica" (Ex-40% funds). The authors wish to thank Ing. G. Mariotti from Mariotti & Pecini, Via S. Pertini 41, 50019 Sesto Fiorentino (Firenze), Italia for his technical support.

Notation

A = the cross section for recirculation, m^2
B = baffle width, m
D = stirrer diameter, m
g = acceleration of gravity, m/s^2
L = width to be used in Eq. A4, m
N = stirrer speed, 1/s
P_g = gassed power consumption, W
P_o = ungassed power consumption, W
Q_{gv} = gas-flow rate, m^3/s
Q_l = recirculated gas-flow rate into the lower turbine, m^3/s
Q_m = mixture flow rate, m^3/s
Q_r = recirculated gas-flow rate, m^3/s
Q_u = recirculated gas-flow rate into the upper turbine, m^3/s
T = vessel diameter, m
V_0 = mean velocity to be used in Eq. A1, m/s
V_1 = mean velocity to be used in Eq. A1, m/s
V_b = bubble rising velocity in a quiescent liquid, m/s
V_g = effective gas velocity, m/s
V_l = effective liquid velocity, m/s
V_m = mean velocity, m/s
V_m^* = mean velocity to be used in Eq. A1, m/s
β_t = cavities gas holdup (Eq. 19)
ΔC = clearance between impellers, m
ΔM = specific energy, m^2/s^2
ϵ = mean gas holdup into the tank
ϵ_m = mean gas holdup of the mixture flowing in the turbine
ϵ_t = mean gas holdup inside the turbine
μ = viscosity, kg/m·s
μ_{mbc} = mechanical efficiency
π = 3.141592..
ρ = density, kg/m^3
ρ_m = mean density in the tank, kg/m^3
ρ_t = mean density of the mixture into the turbine, kg/m^3
σ = surface tension, N/m

Subscripts

g = gas phase
l = liquid phase

Dimensionless number

Eo_{crit} = Eotvos number = $[d_b^2 \cdot (\rho_l - \rho_g) \cdot g]/\sigma$
Fl = flow number (Eq. 8)
Fl_G = gas-flow number $= Q_{gv}/(N \cdot D^3)$
M = fluid properties group (Eq. 26)

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Appendix

Gas holdup correlation

According to Paglianti et al. (2000), the value of the gas holdup correlation in stirred-tank reactors equipped with Rushton turbine can be evaluated as

$$\epsilon = \frac{V_m^* - V_0}{V_m^* + V_1}, \tag{A1}$$

where

$$V_m^* = \frac{Q_{gv} + 0.04 \cdot N \cdot D^3}{\pi \cdot \dfrac{T^2}{4}} \tag{A2}$$

$$V_0 = V_b \cdot \left[0.0605 \cdot \left(\frac{D}{T} \right)^2 \right], \tag{A3}$$

$$V_1 = V_b \cdot \left[\frac{(T - 2 \cdot L)^2 - 1.21 \cdot D^2}{T^2} \right], \tag{A4}$$

and

$$L = (B + 0.027). \tag{A5}$$

Manuscript received Oct. 17, 2000, and revision received May 18, 2001.